Hydraulics

→ 1 poise = 10^{-1} Ns/m² ⇒ Dynamic viscosity

→ 1 stoke = 10^{-4} m²/s ⇒ kinematic viscosity

→ $Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$

→ $h_f = \frac{32\mu VL}{\gamma D^2} = \frac{32\nu VL}{gD^2} = \frac{64V^2L}{Re\,2gD}$

→ $\Delta p = \gamma h_f$ ⇒ for horizontal pipe

→ $\frac{\Delta(p+\gamma z)}{L} \Leftarrow \left(-\frac{\partial p}{\partial x}\right)$ (*)

→ $\tau_0 = -\frac{\partial p}{\partial x}\,\frac{R}{2}$

→ $\nu = \frac{\mu}{\rho}$

→ $\tau = \tau_0 \frac{r}{R}$

→ $\tau = -\frac{\partial v}{\partial r}$

→ $\frac{p_1-p_2}{L} = \left(-\frac{\partial p}{\partial x}\right)$

→ $v = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)(R^2-r^2)$

→ $v_{max} = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$

→ $v_{avg} = \frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2$

→ $v_{avg} = \frac{1}{2} v_{max}$

→ $Q = \frac{\pi}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^4$

· Turbulent flow

→ $f = \frac{8\tau_0}{\rho V^2}$ f = friction factor

→ $h_f = \frac{fLV^2}{2gD}$ (Darcy's Weisbach eqn for friction loss)

→ $h_f = \frac{4f'LV^2}{2gD}$ (f' = fanning coefficient)

→ $f = \frac{8V_*^2}{V^2}$ (*)

→ $\frac{V_*}{V} = \sqrt{f/8}$ → $V_* = \sqrt{\frac{\tau_0}{\rho}}$

from Nikuradse's Experiment

(a) $\frac{V_* k}{\nu}$

$\frac{V_* k}{\nu} \le 3$ ⇒ smooth

$3 < \frac{V_* k}{\nu} < 70$ ⇒ transition

$\frac{V_* k}{\nu} \ge 70$ ⇒ rough

(b) (k/δ')

$(k/\delta') \le 0.25$ ⇒ smooth

$0.25 < (k/\delta') < 6.0$ ⇒ transition

$(k/\delta') \ge 6.0$ ⇒ rough

(c)

$\frac{Re\sqrt{f}}{R/k} \le 17$ ⇒ smooth

$\frac{Re\sqrt{f}}{R/k} > 400$ ⇒ rough

Boussinesq's theory

$\tau = \mu\frac{d\bar v}{dy} + \eta\frac{d\bar v}{dy}$

It is difficult to predict η (rarely used)

μ = viscosity

η = eddy viscosity

$\bar v$ = mean velocity

ε = kinematic viscosity

friction factor

(a) Laminar flow

$f = \frac{64}{Re}$

(b) smooth pipe

→ $f = \frac{0.316}{Re^{1/4}}$ Blasius eqn

($Re \le 10^5$) सत्य मान

→ $f = 0.0032 + \frac{0.221}{Re^{0.237}}$

($Re > 10^5$) देखि

→ main equation

$\frac{1}{\sqrt f} = 2.0\log_{10}(Re\sqrt f) - 0.8$

(c) for rough pipe

$\frac{1}{\sqrt f} = 2.0\log_{10}(R/k) + 1.74$

(d) Colebrook equation

$\frac{1}{\sqrt f} = -2.0\log_{10}\left(\frac{k/D}{3.7} + \frac{2.51}{Re\sqrt f}\right)$

(e) Commercial pipes

$\frac{1}{\sqrt f} - 2.0\log_{10}(R/k) = 1.74 - 2.0\log_{10}\left(1 + 18.7\cdot\frac{R/k}{Re\sqrt f}\right)$

Reynold's theory

$\tau = \rho\,\overline{u_x u_y}$

u_x = fluctuating component of velocity in x-dirⁿ due to turbulence

u_y = in y direction

ludwing prandtl's mixing length theory

$$\tau = \mu \frac{d\bar{v}}{dy} + \rho l^2 \left(\frac{dv}{dy}\right)^2$$

from nicurade's experiment

$$l = 0.4y - 0.44\left(\frac{y^2}{R}\right)$$

→ $\frac{V_{max} - V_{random}}{V_*} = 5.75 \log_{10}(R/y)$

→ $\frac{V_{random}}{V_*} = 5.75 \log_{10}(y/K) + 8.5$

$$\frac{dv}{dy} = \frac{2.5 V_* K}{y}$$

(velocity distribution in rough pipe valid for whole region)

→ $\frac{V_{max}}{V_{mean}} = 1.33\sqrt{f} + 1$

→ $\frac{V_*}{V_{mean}} = \sqrt{f/8}$ (*)

→ $\frac{\Delta p}{L} = \frac{4\tau_0}{D}$

velocity distribution in terms of mean velocity

$\frac{v}{V_*} = 5.75 \log_{10}\left(\frac{V_* R}{\nu}\right) + 1.75$ (for smooth)

$\frac{v}{V_*} = 5.75 \log_{10}(R/K) + 4.75$ (for ~~smooth~~ rough)

→ identical equation

$$\frac{v}{V_*} = 5.75 \log_{10}\left(\frac{V_* y}{\nu}\right) + 5.5$$

→ $\frac{V_{max} - V_{mean}}{V_*} = 3.75$

thickness of laminar sub layer

$$\delta' = \frac{11.6\nu}{V_*}$$

$\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) = h_f$

(inclined pipe)

→ $\tau_0 = -\frac{d(p + \gamma z)}{dx} R/2$

→ mass flow rate (m'/t)

$$= \frac{m'}{time}$$

→ volumetric flow rate (Q)

$$= \frac{m'}{\rho}$$

→ radial distance at which the mean velocity occur

$$= 0.707R$$

→ inclined हु गर्दा use Bernoulli's equation to calculate different variables

→ power $(P) = \frac{Q\Delta p}{\eta}$

(required by machine if it's η)

→ pump दिएको section मा head of pump add गर्नु पर्छ

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_L$$

head of pump — head loss

→ $\tau_0 = \frac{\rho v^2 f}{8}$

Question trick

→ $\Delta p, L$ दिएको हु गर्दा smooth rough छुट्याउने गर्दा $\tau_0 = -\frac{\partial p}{\partial x} R/2$ बाट τ_0 निकाल्ने

$$V_* = \sqrt{\frac{\tau_0}{\rho}}$$

अनि $\frac{V_* K}{\nu}$ बाट smooth / rough निकाल्ने

→ SI unit of

$$-\frac{\partial p}{\partial x} = N/m^2/m$$

→ max^m flow in laminar

$$\frac{Re}{Re_{max}} = \frac{Q}{Q_{max}}$$

Re and Q Given

$Re_{max} = 2000$

→ maximum velocity occur at centre

center line velocity = v_{max}

Minor head loss

→ due to sudden enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}$$

due due sudden contraction

C_c = Coefficient of contraction

$$\therefore h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2$$

entry loss

$= 0.5 \frac{V_1^2}{2g}$ | exit loss $\frac{V_2^2}{2g}$

Type of loss

→ entry loss + head loss + sudden exp^n + sudden contraction + Exit loss

Pipe flow problem

$Q.\ H = \frac{fLV^2}{2gD}$

$H = \frac{8fLQ^2}{\pi^2 g D^5}$

$\rightarrow H = rQ^2$

$\rightarrow r = \frac{8fL}{\pi^2 g D^5}$

pipe in series

$\rightarrow Q_1 = Q_2 = Q_3$

$\rightarrow H = h_{f1} + h_{f2} + h_{f3}$

$\rightarrow H = \frac{f_1 L_1 V_1^2}{2gD_1} + \frac{f_2 L_2 V_2^2}{2gD_2} + \dots$

$\rightarrow rQ^2 = r_1 Q^2 + r_2 Q^2 + \dots$

$\rightarrow r = r_1 + r_2 + \dots$

$\rightarrow \frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \dots$

pipes in parallel

$\rightarrow h_{f1} = h_{f2} = h_{f3}$

$\rightarrow Q = Q_1 + Q_2 + Q_3 + \dots$

$\rightarrow \frac{1}{\sqrt{r}} = \frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}} + \dots$

$\rightarrow \sqrt{\frac{D_e^5}{L_e}} = \sqrt{\frac{D_1^5}{L_1}} + \sqrt{\frac{D_2^5}{L_2}} + \dots$

$\rightarrow Q_2 = \sqrt{\frac{r_1}{r_2}}\ Q_1$

$\rightarrow Q_3 = \sqrt{\frac{r_1}{r_3}}\cdot Q_1$

or $r_1 Q_1^2 = r_2 Q_2^2 = r_3 Q_3^2$

Hardy cross method table

LOOP-I

pipe	length (L)	$r = \frac{8fL}{\pi^2 g D^5}$	Q	rQ^2	$\lvert 2rQ \rvert$	$\Delta Q = \frac{\Sigma rQ^2}{\Sigma \lvert 2rQ \rvert}$
AB						
BC						
				ΣrQ^2	sum $\lvert 2rQ \rvert$	

Here $rQ = h_f$

$\Delta Q = \frac{\Sigma rQ^2}{\Sigma \lvert 2rQ \rvert}$

clockwise loop taken as (+ve)

anticlockwise (−ve)

Analysis of water hammer

(a) pressure rise due to water hammer for gradual closure of valve i.e $t > 2L/c$

(i) $F = \rho A L \times \frac{V}{t} \Rightarrow m \times a$

axial force acting due to retardation

(ii) F due to pressure wave $= p \times A$

$\therefore \rho A L \times \frac{V}{t} = p \times A$

$\rightarrow \therefore p = \frac{\rho L V}{t}$

$\rightarrow$ so, $h = \frac{LV}{gt}$

(b) instantaneous closure of valve in rigid pipe $t < \frac{2L}{c}$

Loss in KE = Gain in strain energy

$\frac{1}{2} m v^2 = \frac{1}{2}$ stress $\times$ strain $\times$ volume

$\frac{1}{2}\rho A L V^2 = \frac{1}{2} \times \frac{p^2}{K} AL$

$p = V\sqrt{\rho K}$

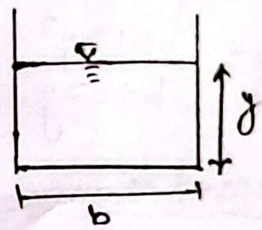
$\boxed{\therefore p = \rho V c}$ $\quad c = \sqrt{K/\rho}$

instantaneous closure of valve in elastic pipe

$p = \sqrt{\frac{\rho V^2}{\left(\frac{1}{K} + \frac{D}{Et}\right)}} = V\sqrt{\frac{\rho}{\left(\frac{1}{K} + \frac{D}{Et}\right)}}$

Geometrical properties of

(a). of rectangular shaped open channel flow



$\rightarrow$ Area (A) = by

$\rightarrow$ wetted perimeter $P = b + 2y$

$\rightarrow$ hydraulic radius (R)

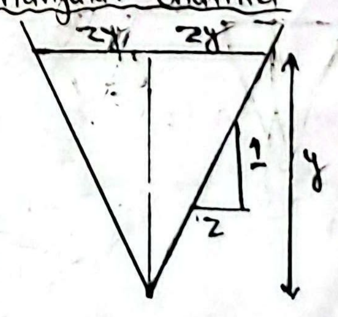
$R = A/P = \frac{by}{b+2y}$

$\rightarrow$ Top width (T) = b

$\rightarrow$ Hydraulic depth (D) = A/T

$\therefore D = \frac{by}{b} = y$

(b) Triangular channel



$A = zy^2$

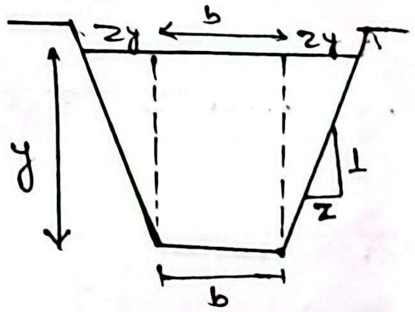
$P = 2y\sqrt{z^2+1}$

$R = A/P = \frac{zy^2}{2y\sqrt{z^2+1}}$

$T = 2zy$

$D = A/T = \frac{zy^2}{2zy} = y/2$

Trapezoidal Channel



$A = by + zy^2$

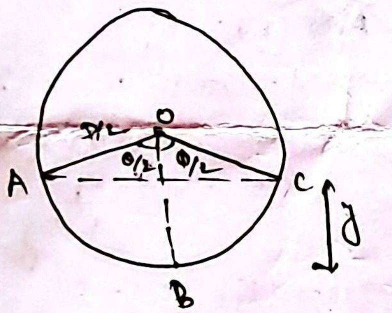
$P = b + 2y\sqrt{z^2+1}$

$R = A/P = \dfrac{by+zy^2}{b+2y\sqrt{z^2+1}}$

$T = b + 2zy$

$D = A/T = \dfrac{by+zy^2}{b+2zy}$

Circular channel



$\cos\theta/2 = \left(\dfrac{D/2 - y}{D/2}\right)$

$\theta/2 = \cos^{-1}\left(\dfrac{D/2-y}{D/2}\right)$

→ flow area (A)

$A = \dfrac{D^2}{4} \times \dfrac{\theta}{2\pi} - 2 \times \frac{1}{2} \times \frac{D}{2} \times \sin\theta/2 \times D/2 \cos\theta/2$

$= \dfrac{D^2\theta}{8} - \dfrac{D^2}{8}(2\sin\theta/2 \cos\theta/2)$

$= \dfrac{D^2}{8}(\theta - \sin\theta)$

$T = D\sin\theta/2$

$P = D\,\theta/2$

छिद्रों के गुणे

$Re = \dfrac{\rho V D}{\mu} = \dfrac{\rho V R}{\mu}$

D → Dia

R → Hydraulic radius

$fr = \dfrac{V}{\sqrt{gD}} = \dfrac{V}{\sqrt{gy}}$

D → Hydraulic depth

$fr > 1$ super critical flow

$fr = 1$ critical flow

$fr < 1$ sub critical flow

$Re < 500$ → laminar

$500 < Re < 12500$ → transition

$12500 < Re$ → turbulent

Uniform flow equation

(a) from chezy's equation

$V = C\sqrt{RS}$

(b) from manning's equation

$V = \frac{1}{n} A R^{2/3} S_0^{1/2}$

#

Chezy's constant :

(a) Kutter's formula

$$C = \dfrac{23 + \dfrac{0.00155}{S} + \frac{1}{n}}{1 + \left(23 + \dfrac{0.00155}{S}\right)\dfrac{n}{\sqrt{R}}}$$

(b) Bazin formula

$$C = \dfrac{157.6}{1.81 + \dfrac{m}{\sqrt{R}}}$$

(c) manning's formula

$$C = \frac{1}{n} R^{1/6}$$

Equivalent roughness

(a) Horton and Einstein Eq^n

$$n = \left[\dfrac{\sum_{i=1}^{N} P_i n_i^{3/2}}{P}\right]^{2/3}$$

(b) pavlovski's equation

$$n = \left[\dfrac{\sum_{i=1}^{N} P_i n_i^2}{P}\right]^{1/2}$$

(c) Lotter equation

$$n = \dfrac{P R^{5/3}}{\sum_{i=1}^{N}\left(\dfrac{P_i R_i^{5/3}}{n_i}\right)}$$

most Efficient

rectangular channel

$p = b + 2y$

for most efficient 'p' minimum

$\therefore \dfrac{dp}{dy} = -\dfrac{A}{y^2} + 2 = 0$

$A = 2y^2$

$by = 2y^2$

$b = 2y$

$\rightarrow b = 2y$

so, $R = y/2$

(b) triangular section

$\rightarrow A = zy^2$

$\rightarrow p = 2y\sqrt{z^2+1}$

$\therefore y = \sqrt{\dfrac{A}{z}}$ — (a)

$\therefore p = 2\sqrt{\dfrac{A}{z}} \cdot \sqrt{z^2+1}$

$p^2 = 4\dfrac{(z^2+1)}{z} A = \left(4z + \dfrac{4}{z}\right)A$

$\therefore 2p \cdot \dfrac{dp}{dz} = \left(4 - \dfrac{4}{z^2}\right)A = 0$

$z = 1$

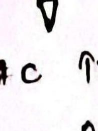
$\therefore R = A/p = \dfrac{zy^2}{2y\sqrt{z^2+1}}$

put $z = 1$

so, $R = \dfrac{y}{2\sqrt{2}}$

so, slope $z:1 = 1:1$

$\theta = 90°$



c Trapezoidal channel

$R = A/p = \dfrac{(2y\sqrt{z^2+1} - 2zy + zy)y}{2y\sqrt{1+z^2} - 2zy + 2y\sqrt{1+z^2}}$

$R = y/2$

continue ---

$\dfrac{dp}{dz} = -y + \dfrac{2y}{2\sqrt{z^2+1}} \cdot 2z = 0$

$2z = \sqrt{1+z^2}$

$\therefore z = \dfrac{1}{\sqrt{3}}$

$\theta = 60°$

$\therefore$

summary

(a) Rectangular channel

$\rightarrow b = 2y$

$\rightarrow R = y/2$

(b) triangular channel

$z:1 = 1:1$

$\theta = 90°$

$R = \dfrac{y}{2\sqrt{2}}$

(c) trapezoidal

$H:V = 1:\sqrt{3}$

$\theta = 60°$

$R = y/2$

(d) circular section

$\rightarrow$ for max^m discharge

$y = 0.95D$ → Dia of pipe

$R = 0.29D$

$\rightarrow$ for max^m mean velocity

$y = 0.81D$ → Dia of pipe

$R = 0.30D$

Energy and momentum principle

$\rightarrow$ Total Head $\rightarrow H = y + z + \dfrac{v^2}{2g}$

Pressure head, datum head, velocity head

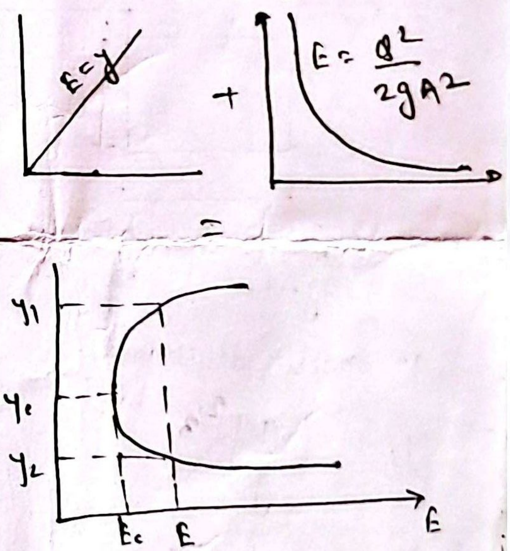
$\rightarrow$ specific energy (E)

(bed slope = datum given)

$E = y + \dfrac{v^2}{2g}$ — (a)

$E = y + \dfrac{Q^2}{2gA^2}$ — (b)

$\rightarrow$ specific energy curve



y_1 and y_2 alternate depth

$\rightarrow$ condition for critical energy

$\dfrac{Q^2 T}{g A^3} = 1$

$\rightarrow \dfrac{Q^2}{g} = \dfrac{A^3}{T}$

Condition for critical energy

→ $\boxed{\frac{Q^2}{g} = \frac{A^3}{T}}$

$\frac{Q^2 T}{g A^3} = 1$

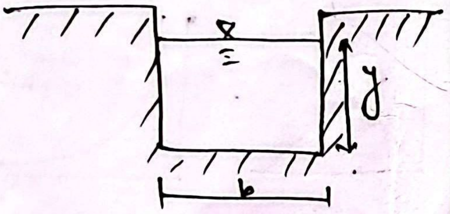
→ $\boxed{fr = 1}$

Critical Condition

$y_1 > y_c \Rightarrow$ subcritical flow

$y_2 < y_c \Rightarrow$ supercritical flow

Specific energy for a rectangular channel



→ $E = y + \frac{Q^2}{2gA^2} = y + \frac{q^2}{2gy^2}$

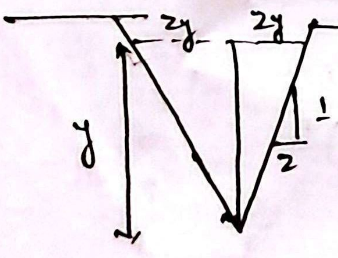
q = specific discharge

→ $y_c = \left(\frac{q}{g}\right)^{2/3}$

→ $E_{min} = 1.5 y_c$

→ $Q = \frac{1}{n} A R^{2/3} S_0^{1/2}$

Specific energy for triangular portion



$T = 2zy$

$A = zy^2$

$R = A/P$

Continue....

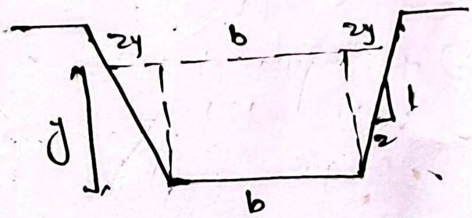
→ $y_c = \left(\frac{2Q^2}{g z^2}\right)^{1/5}$

→ Critical velocity

$V = \sqrt{g \times y_c/2}$

→ $E_{min} = 4/5\, y_c$

Trapezoidal channel:



$P = b + 2zy$

$A = by + zy^2$

→ Condition for critical velocity

$\frac{Q^2 T}{g A^3} = 1$

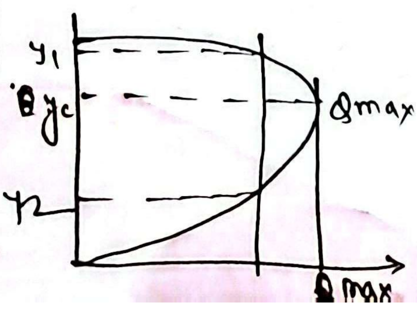
$\frac{Q^2}{g} = \frac{A^3}{T}$

~~Depth~~ Discharge depth curve for given specific energy

$E = y + \frac{Q^2}{2gA^2}$

$Q = A\sqrt{2g(E-y)}$ — (x)

(Constant put)



Condition for maxm discharge

→ $\frac{dQ}{dy} = 0$

→ $E - y = \frac{A}{2T}$

Hump wall आके पढे

Equation for gradually varied flow

$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}} = \frac{S_0 - S_f}{1 - fr^2}$

Dynamic eqn for GVF in wider rectangular channel

→ from manning's formula

$\frac{dy}{dx} = S_0 \frac{1 - (y_n/y)^{10/3}}{1 - (y_c/y)^3}$

– from chezy's formula

$\frac{dy}{dx} = S_0 \frac{1 - (y_n/y)^3}{1 - (y_c/y)^3}$

steep slope

$S_0 > S_c$ $y_n < y_c$

critical slope

$S_0 = S_c$ $y_n = y_c$

mild slope

$S_0 < S_c$ $y_n > y_c$

Horizontal

$S_0 = 0$ $y_n = \infty$

adverse

$S_0 < 0$ $y_n = $ Img.